Incorporating the Effects of Traffic Signal Progression Into the Proposed Incremental Queue Accumulation (IQA) Method

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Abstract

A new approach for computing the uniform delay component based on incremental queue accumulation (IQA) has been presented in a companion paper to this one. The method is fully consistent with the intent of Webster’s first delay term, and under the same assumptions as Webster’s, produces identical results. The method provides added flexibility beyond what the purely analytical approach can handle, particularly in the complex cases of protected-permitted turns, two green times per cycle, and the effect of sneakers, among others. The companion paper provides analytical details of the methods, and numerical examples to illustrate its applications.

This paper focuses on incorporating the effects of signal coordination (or lack thereof) on the uniform delay component in the context of the IQA approach. It is shown that the IQA method is fully capable of handling variable arrival rates in different parts of the cycle, and that under similar assumptions as the Highway Capacity Manual (HCM), would produce consistent results as well. An important finding in the course of this investigation was that the current progression adjustment factor in the HCM is based on the simplifying assumption that the uniform queue dissipates at the same point in the cycle, regardless of the level of coordination. Clearly this is an erroneous assumption that the IQA is now able to overcome.

Taken together, the findings in this and the companion paper have outlined a rational, consistent and flexible uniform delay estimation procedure that is far superior to the current HCM approach. The approach has been endorsed by the Signalized Intersection Subcommittee (SigSub) of the TRB Highway Capacity Committee.
Background and Problem Statement

In January, 2005 the Signals Subcommittee (hereafter referred to as SigSub) of the TRB Highway Capacity and Quality of Service Committee (AHB40) approved in principle a proposed new method for evaluating the $d_1$ component of the delay equation (Eq. 16-12) in the 2000 Highway Capacity Manual (HCM) [1]. Rather than calculate the formula for the area of the triangle which this equation represents, the area is incrementally accumulated by calculating the areas of time slices of the queue accumulation polygon using a method called Incremental Queue Accumulation (IQA).

The fundamentals of the method are described in detail in a companion paper submitted to TRB by the authors [2]. The essence of this approach was also presented at the January, 2005 SigSub meeting in a white paper titled “Proposed New Calculation Method for Existing HCM Delay Procedures” [3]. The main motivation for the white paper was that a number of limiting assumptions which are required by the ‘formula’ approach can be lifted using the ‘incremental’ approach, thereby making the results more understandable, accurate and applicable for a wider range of typical conditions, without changing the basic premise of the calculations.

The IQA method, as initially presented, suggests the use of equal-sized time slices, adding and subtracting the number of arrivals and departures during each time slice to the queue at the start of the time slice and resulting in the queue at the end of the time slice. Since the queue during a one-second time slice is numerically equal to the $D_1$ delay during that time slice, simply adding up all the products of queue and time slice durations for all time slices results in the total $D_1$ delay experienced in a cycle. Dividing that value by all arrivals in the cycle yields the average $d_1$ delay term. The method also illustrated how the maximum back of queue ($Q_1$) can be accumulated in a similar manner. In the final analysis, it is illustrated how the IQA method can be simplified to a calculation of trapezoids which represent the periods of time during the cycle where the inflow and outflow rates are invariant. In essence, the IQA method is a more generalized approach to calculating the queue accumulation area using multiple trapezoids, while the original Webster formula is a degenerate case with two degenerate trapezoids which form a single triangle whose area is $D_1$.

A major unresolved point after the January 2005 meeting was how to handle the effects of progression in the IQA method. The white paper proposed an approach which was not detailed, and substantial discussion resulted. The focus of this paper is to provide a resolution of the progression issue, with a very straightforward solution that is again 100% compatible with the intent of the 2000 HCM, and at the same time resolves a previously unknown problem related to the original progression adjustment factor (more on this later). Several additional issues related to the IQA implementation, such as carrying out lane-by-lane analysis are addressed in the discussion section of the paper.

The paper is organized as follows. First, a review of the progression factors in the various HCM editions is presented, followed by a proposed correction to the current HCM progression factor and a discussion of its implications on the delay model. Three numerical examples using the IQA method with platooned arrivals are presented next, followed by a set of conclusions and recommendations for future research.
Derivation of the Progression Factor Method of the HCM

The 2000 HCM [1] and all versions of the HCM starting with the 1985 HCM [4] in one form or another, addresses the effect of progression on delay by multiplying the delay calculated for uniform arrivals by a progression factor (PF) determined by the following formula (2000 HCM Eq. 16-11):

\[
PF = \frac{(1 - P) f_{PA}}{(1 - g/C)} \quad \text{Eq. 1}
\]

where:

- \( P \) = proportion of vehicles arriving on green (0.0 – 1.0)
- \( f_{PA} \) = supplemental adjustment factor for early or late platoon arrivals
- \( g/C \) = effective green-to-cycle ratio for the movement
- \( R_p \) = platoon ratio defined as \( P/(g/C) \)

To accommodate conditions where \( P \) values are not known, the HCM describes six different arrival types numbered 1 through 6 for various qualities of progression, and defines a range of platoon ratios, \( R_p \), which represent each arrival type, with an average platoon ratio for each. For uniform arrivals, \( P = g/C \), so \( R_p = 1 \). Using the average platoon ratio for a given arrival type, \( P \) can be calculated by inverting the definition of \( R_p \) above \([P = R_p(g/C) \leq 1.0]\) and this \( P \) value can be used in the PF formula above (using a prescribed value of \( f_{PA} \) for each arrival type).

In the 1994 HCM [5] update, the PF adjustment was removed from the incremental delay term, \( d_2 \) and was applied only to the \( d_1 \) term, and the \( f_{PA} \) factor was added. In HCM 2000, a more precise PF formula was introduced in support of the HCM queue model (PF\(_2\)), but no change was made to the PF formula used for delay. What is not commonly understood is that the PF\(_2\) formula is an adjustment to the original PF which was needed in order to calculate the correct value of the back of queue for platooned arrivals, with queue dissipation times which vary depending on the degree of platooning. The original PF formula for delay (which was originally applied to both \( d_1 \) and \( d_2 \)) assumes that the queue dissipation time is the same under all signal progression scenarios, and thus likely overestimates the delay for good progression and underestimates delay for poor progression (as can be seen later). The rather arbitrary \( f_{PA} \) adjustment in the 1994 PF definition was an attempt to correct for the possibility of early and late platoon arrivals under some, but not all, arrival type conditions [6], and further complicates the effect of the erroneous queue dissipation time since \( f_{PA} \) does not reduce the queue dissipation error, but in some cases, could actually exacerbate it.

The underlying theory for both PF and PF\(_2\) actually assumes two different flow rates, one each for the effective red and green periods (i.e. \( V_r \) during the effective red time \( r \) and \( V_g \) during the effective green time \( g \)), where the weighted average cycle flow rate is the analysis flow rate \( V \), and the ratio of the arrivals during green to the total arrivals is the \( P \) value. Thus:

\[
V = \frac{(V_r r + V_g g)}{C} \quad \text{Eq. 2}
\]

\[
P = \frac{V_g g}{(V C)} \quad \text{Eq. 3}
\]

From Eq. 3, it can be seen that \( V_g \) can be calculated from the average volume when the \( P \) and \( g/C \) values are known, and that \( V_r \) can be then calculated from Eq. 2 by substituting for the \( V_g \) value.
Thus, for a given $P$, or a $P$ estimated from a given arrival type and average platoon ratio, the flow rates during red and green can be calculated as:

\[
V_g = \frac{V P}{(g/C)} \quad \text{Eq. 4}
\]

\[
V_r = \frac{(V C - V_g g)}{r} = \frac{V (1 - P)}{(1 - g/C)} \quad \text{Eq. 5}
\]

The HCM formula for PF is intended to be simply the ratio of the $d_1$ delays with and without the different flow rate assumptions (as we’ll see later, and disregarding the $f_{PA}$ adjustment for the moment): the numerator assumes the platooned flow rates $V_g$ and $V_r$ and the denominator assumes the average flow rate $V$. In order to compute this ratio, a major simplifying assumption is made -- that the queue dissipation time is the same for both platooned and uniform arrivals. (In reality, the queue will dissipate earlier under good progression and later under poor progression.)

For either case, it can be shown that the $d_1$ delay term is equal to one-half the queue at the start of green (end of red) multiplied by the sum of the red time plus the assumed time of queue dissipation, divided by the total arrivals per cycle. Thus, for the uniform arrival case:

\[
d_1 = 0.5 \frac{r V (r + g_q)}{VC}, \quad \text{Eq. 6}
\]

where $g_q$ is the assumed time of queue dissipation

For the HCM’s simplified platooned arrival case (where $g_q$ is assumed to be the same as above)

\[
d_1' = 0.5 \frac{r V_r (r + g_q)}{VC} \quad \text{Eq. 7}
\]

Thus, by definition and substitution with the equations above:

\[
PF = \frac{d_1'}{d_1} = \frac{V_r V}{V} = \frac{(1 - P)}{(1 - g/C)} \quad \text{Eq. 8}
\]

Except for the ambiguous $f_{PA}$ first found in the 1994 HCM equation for PF which was based on FHWA research by Fambro et al [6], Eq. 8 is the same formula for PF, thus illustrating that the intent of the PF adjustment in the HCM is to account for the differing flow rates during red and green on $d_1$ delay, and that these differing flow rates can be easily computed when $P$ or $R_p$ are known (or assumed).

**Proposed Change to the HCM Progression Adjustment Factor**

The correct derivation of $g_q$ for the platooned case ($g_q'$) requires the proper estimation of the time it takes to dissipate the accumulated arrivals during effective red ($r * V_r$) at the net departure rate during the effective green ($s - V_g$). Thus:

\[
r V_r = g_q' (s - V_g) \quad \text{Eq. 9}
\]

Solving for $g_q'$ yields:

\[
g_q' = \frac{r V_r}{(s - V_g)} \quad \text{Eq. 10}
\]
and the \( d_1 \) delay term using the correct queue dissipation time becomes:

\[
d_1' = 0.5 \frac{r}{V_r} \frac{r + g}{VC} \quad \text{Eq. 11}
\]

Again, by definition, the correct definition of PF for the \( d_1 \) term (labeled here as \( \text{PF}_1 \)) is the ratio of \( d_1' \) (Eq. 11) to \( d_1 \) (Eq. 6), which, with substantial algebraic simplification yields:

\[
\text{PF}_1 = \frac{(1 - R_p g / C) (1 - V / s)}{(1 - g / C) (1 - R_p V / s)} \left[ 1 + \frac{V}{s} \frac{(1 - R_p)}{(1 - g / C)} \right] \quad \text{Eq. 12}
\]

The formulation of the progression factor in Eq. 12 permits a precise calculation of the effect of progression on delay, using the same underlying assumptions of the current HCM, but without the simplifying assumption of constant queue dissipation time or the rather arbitrary use of \( f_{PA} \). From this point forward Eq. 12 should be perceived as the correct formula for PF in the HCM, if a formula is to be used, with the recognition that the use of the \( f_{PA} \) factor has been eliminated. Note that the value of \( R_p \) used in Eq. 12 must be consistent with the requirement that \( P = R_p g / C \leq 1.0 \) (e.g., calculate \( R_p = P / (g / C) \) for the value of \( P \) actually used).

It is interesting to note that the first part of the \( \text{PF}_1 \) equation is the original PF formula (without the \( f_{PA} \) factor). The second part of the \( \text{PF}_1 \) equation is the same correction applied to PF in the calculation of \( \text{PF}_2 \) for the HCM first-term queue, \( Q_1 \), (see [1] Eq. G16-8) which is the correction necessary for the proper heights of the queue accumulation triangles. This leaves the third part as the correction necessary for the proper areas of the queue accumulation triangles.

**Implications of the \( \text{PF}_1 \) Factor when Implementing the IQA Method**

The \( V_g \) and \( V_r \) values are the flow rates which can be used for platooned arrivals in the IQA method in place of the average flow rate, and except for the differences between PF and \( \text{PF}_1 \) described above, will generate exactly the same results as applying the progression factor. In other words, if the more precise \( \text{PF}_1 \) were used in the 2000 HCM delay formula instead of the original PF, a more accurate delay calculation would be made for platooned arrivals (due to the correct assessment of queue dissipation time), and using the \( V_g \) and \( V_r \) values from Eqs. 4 and 5 in the IQA method will generate exactly the same results as when \( \text{PF}_1 \) is used (except for the possible effect of \( f_{PA} \)).

The \( f_{PA} \) factor in the HCM appears to be a limited, incomplete attempt to account for early and late platoon arrivals for only two of the six arrival types, and having prescribed constant values in the HCM, does not go far enough to describe all of the potential early/late platoon arrivals that can exist for all arrival types. Indeed, it actually assumes a specific, arbitrary late arrival scenario for arrival type 4 and a specific, arbitrary early arrival scenario for arrival type 2, with no real basis for a user to change this assumption for these arrival types, or to apply similar assumptions to other arrival types. Since the IQA method permits the explicit definition of any early or late platoon arrival condition for any analysis condition, the application of the 1994 \( f_{PA} \) factor for a limited number of arrival type cases would be inconsistent, and thus has been deleted from the proposed procedure. It is left to the discretion of the analyst to define an appropriate arrival pattern that accurately reflects the platoon arrival conditions.
Thus, when in a typical capacity analysis P is either known or estimated from the arrival type, the same assumptions built into the PF, PF$_1$ and PF$_2$ formulas can be used to estimate $V_g$ and $V_r$ for the IQA method, and thus calculate the same results without the explicit adjustment of $d_1$ by a PF factor of any kind. The extension of this, however, allows known values of $V_g$ and $V_r$ to be used, and in the most extreme case when a measured or estimated arrival pattern is known throughout the cycle, even to a second-by-second precision, this arrival pattern can be used as the inflow portion of the IQA, with even better results. (The demand arrival pattern, should be established by taking flow rate measurements at the upstream end of the intersection approach, away from the effects of any queues that may have developed.) These results can be expected to compete favorably with the accuracy of other, much more sophisticated traffic flow models, a major step forward for the HCM delay estimation method.

What this means is that another of the constraining, simplifying assumptions of the 2000 HCM can be lifted by use of the IQA method without disturbing the underlying theory of Webster’s original research or the HCM’s application of that theory for platooned arrivals. This can be achieved while at the same time yielding the same (actually better) results when the same assumptions are acceptable, but opening up the possibility of much improved results due to the vastly increased flexibility the IQA method offers. All this can be accomplished with the simple additional calculation of $V_g$ and $V_r$ for use in the IQA method using Eq. 4 and Eq. 5.

**Numerical Examples**

The following are the first two examples described in the companion IQA paper [2] illustrating the application of the IQA method for platooned arrival conditions. In both cases, the PF, PF$_1$ and Q$_1$ calculations are shown, and IQA yields the exact same results for $d_1$ (when PF$_1$ is used) and Q$_1$.

**Example 1 - Simple Movement Controlled by a Simple Red/Green Cycle, Arrival Type 4**

**Given Conditions**

- $\Delta$-increment = 2 sec
- $V = 1800$ vph, or $1800/3600 \times 2 = 1$ veh/increment
- $s = 3600$ vph, or $3600/3600 \times 2 = 2$ veh/increment
- $C = 60$ sec
- $g = 40$ sec

**Computed Results**

- $c = sg/C = 2400$ vph
- $X = V/c = 0.75$
- $d_1 = 0.5C \frac{(1-g/C)^2/(1-Xg/C)}{1-Xg/C} = 6.667$ sec/veh (without PF adjustment)
- $R_p = 1.333$ (assumed for AT=4)
- $f_{PA} = 1.15$ (assumed for AT=4)
- $P = R_p \times g/C = 0.889$
- $PF = (1-P)f_{PA}/(1-g/C) = 0.383 \ (=0.333 \text{ if } f_{PA} \text{ is not used})$
- $d_1*PF = 2.553$ sec/veh
- $MBQ=(\text{maximum back of queue or } Q_1) = 10.0$ using Eq. G16-7 in HCM2000
\[ V_g = 2400 \text{ vph using Eq. 4, or } 2400/3600 \times 2 = 4/3 \text{ veh/increment} \]
\[ V_i = 600 \text{ vph using Eq. 5, or } 600/3600 \times 2 = 1/3 \text{ veh/increment} \]
\[ q_g' = 10 \text{ sec using Eq. 10 (measured from the start of the effective green; or 30 sec from red start)} \]
\[ PF_1 = 0.250 \text{ using Eq. 12} \]
\[ d_1 \times PF_1 = 1.667 \text{ sec/veh} \]

The results are summarized in tabular form in Table 1, and shown in graphical form in Figure 1.

**Table 1. IQA Method Applied to Platoon Arrivals: Example 1**

<table>
<thead>
<tr>
<th>Δ#</th>
<th>Time (sec)</th>
<th>#In</th>
<th>#Out</th>
<th>IQA</th>
<th>MBQ</th>
<th>IQA*Δ (partial delay)</th>
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<td>0-2</td>
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<td>2/3</td>
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<td>4-6</td>
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<td>3/3</td>
<td>6/3</td>
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<td>6-8</td>
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<td>4/3</td>
<td>4/3</td>
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</tr>
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<td>5/3</td>
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<td>14/3</td>
<td>16/3</td>
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<td>6/3</td>
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</tr>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
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</tr>
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<td>0</td>
</tr>
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<td>0</td>
</tr>
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<td>50-52</td>
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<td>0</td>
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<td>4/3</td>
<td>0</td>
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<td>-</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>30</strong></td>
<td><strong>30</strong></td>
<td><strong>75/3</strong></td>
<td><strong>30/3</strong></td>
<td><strong>150/3 veh-sec</strong></td>
<td></td>
</tr>
</tbody>
</table>

From Table 1, the first term delay is easily computed as:

\[ d_1' = (150/3)/30 = 1.667 \text{ sec/veh}, \text{ which is identical to the computed value of } d_1 \times PF_1 \]
Figure 1. Illustration of IQA Results for Platooned Arrivals, Example 1

Increment $\Delta = 2$ sec

Each “X” is 1/3 vehicle delayed 2 sec

75 X’s = 150/3 veh-sec

Display:

Increment: 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 3
1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0

IQA (1/3): 1
1 2 3 4 5 6 7 8 9 0 8 6 4 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Example 2 - Permitted Left-Turn with Weighted-Average Saturation flow, Arrival Type 5

Given Conditions
\( \Delta \)-increment = 12 sec
\( V = 300 \text{ vph} \), or \( 300/3600 \times 12 = 1 \text{ veh/increment} \)
\( V_o = 564 \text{ vph} \)
\( s = 600 \text{ vph} \), or \( 600/3600 \times 12 = 2 \text{ veh/increment} \)
\( C = 120 \text{ sec} \)
\( g = 60 \text{ sec} \)

Computed Results
\( c = sg/C = 300 \text{ vph} \)
\( X = V/c = 1.00 \)
\( d_1 = 0.5C(1-g/C)^2/(1-Xg/C) = 30.0 \text{ sec/veh (without PF adjustment)} \)
\( R_p = 1.667 \) (assumed for AT=5)
\( f_{PA} = 1.00 \) (assumed for AT=5)
\( P = R_p * g/C = 0.833 \)
\( PF = (1-P)f_{PA}/(1-g/C) = 0.333 \)
\( d_1*PF = 10.0 \text{ sec/veh} \)
MBQ (maximum back of queue, or \( Q_1 \)) = 10.0 using Eq. G16-7 in HCM2000

\( V_g = 500 \text{ vph} \) using Eq. 4, or \( 500/3600 \times 12 = 5/3 \text{ veh/increment} \)
\( V_r = 100 \text{ vph} \) using Eq. 5, or \( 100/3600 \times 12 = 1/3 \text{ veh/increment} \)
\( g_0' = 60 \text{ sec} \) using Eq. 10 (measured from the start of the effective green; or 120 sec from red start)
\( PF_1 = 0.333 \) using Eq. 12
\( d_1*PF_1 = 10.0 \text{ sec/veh} \)

The results are summarized in tabular form in Table 2, and shown in graphical form in Figure 2.

<table>
<thead>
<tr>
<th>( \Delta # )</th>
<th>time (sec)</th>
<th>#In</th>
<th>#Out</th>
<th>IQA</th>
<th>MBQ</th>
<th>IQAx( \Delta ) (partial delay)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-12</td>
<td>1/3</td>
<td>0</td>
<td>1/3</td>
<td>1/3</td>
<td>4 Effective red phase starts</td>
</tr>
<tr>
<td>2</td>
<td>12-24</td>
<td>1/3</td>
<td>0</td>
<td>2/3</td>
<td>2/3</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>24-36</td>
<td>1/3</td>
<td>0</td>
<td>3/3</td>
<td>3/3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>36-48</td>
<td>1/3</td>
<td>0</td>
<td>4/3</td>
<td>4/3</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>48-60</td>
<td>1/3</td>
<td>0</td>
<td>5/3</td>
<td>5/3</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>60-72</td>
<td>5/3</td>
<td>6/3</td>
<td>4/3</td>
<td>10/3</td>
<td>16 Effective perm. green phase starts</td>
</tr>
<tr>
<td>7</td>
<td>72-84</td>
<td>5/3</td>
<td>6/3</td>
<td>3/3</td>
<td>15/3</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>84-96</td>
<td>5/3</td>
<td>6/3</td>
<td>2/3</td>
<td>20/3</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>96-108</td>
<td>5/3</td>
<td>6/3</td>
<td>1/3</td>
<td>25/3</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>108-120</td>
<td>5/3</td>
<td>6/3</td>
<td>0</td>
<td>30/3</td>
<td>0 Queue dissipates @ time 120</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10</strong></td>
<td><strong>10</strong></td>
<td><strong>25/3</strong></td>
<td><strong>30/3</strong></td>
<td><strong>100 veh-sec</strong></td>
<td></td>
</tr>
</tbody>
</table>

From the table, then \( d_1' = 100/10 = 10.0 \text{ sec/veh} \), which is identical to the \( d_1*PF_1 \) value above.
Figure 2. Illustration of IQA Results for Platooned Arrivals, Example 2

<table>
<thead>
<tr>
<th>X</th>
<th>Increment Δ = 12 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>each “X” is 1/3 vehicle</td>
</tr>
<tr>
<td>X</td>
<td>delayed 12 sec</td>
</tr>
<tr>
<td>X</td>
<td>25 X’s = 300 veh-sec</td>
</tr>
</tbody>
</table>

Display:
```
 r r r r r g g g g g
```
Increment:
```
1 2 3 4 5 6 7 8 9 0
```
IQA (1/3):
```
1 2 3 4 5 4 3 2 1 0
```

It should be noted that in this example both PF and PF\textsubscript{1} are identical. The reason for this is that the movement is operating at capacity (X=1), and therefore, regardless of the arrival type, the queue dissipation time will always be at the end of the effective green time. Had the movement been operating below capacity, discrepancies between the uniform and platooned arrival cases, similar to those shown in Example 1 would have been observed.
Example 3 – Protected-Permitted Left-Turn, Arrival Type 2

Given Conditions
\[ \Delta \text{increment} = 4 \text{ sec} \]
\[ \text{V} = 1800 \text{ vph (2 lanes), or } 1800/3600 \times 4 = 2 \text{ veh/increment} \]
\[ \text{V}_0 = 40 \text{ vph (2 lanes)} \]
\[ s_{\text{prot}} = 3600 \text{ vph, or } 3600/3600 \times 4 = 4 \text{ veh/increment} \]
\[ s_{\text{perm}} = 2700 \text{ vph, or } 2700/3600 \times 4 = 3 \text{ veh/increment} \]
\[ C = 60 \text{ sec} \]
\[ g_{\text{prot}} = 16 \text{ sec} \]
\[ g_{\text{perm}} = 20 \text{ sec} \]

Computed Results
\[ g_q = 0.49 \text{ sec for opposing flow queue dissipation from HCM procedures} \]
\[ d_1 \text{ from HCM procedures} = 10.2 \text{ sec/veh (without PF adjustment)} \]
\[ R_p = 0.667 \text{ (assumed for AT=2)} \]
\[ f_{\text{PA}} = 0.93 \text{ (assumed for AT=2)} \]
\[ P = R_p \times g/C = 0.178 \text{ (for protected phase only, per HCM guidelines page 16-20)} \]
\[ \text{PF} = (1-P)f_{\text{PA}}/(1-g/C) = 1.043 \text{ (=1.121 if } f_{\text{PA}} \text{ is not used)} \]
\[ d_1 \times \text{PF} = 10.6 \text{ sec/veh} \]
\[ \text{MBQ (maximum back of queue, or } Q_1) = 14.7 \text{ using Eq. G16-7 in HCM2000} \]

For IQA comparison, use weighted-average \( s = 3100 \text{ vph and combined } g = 36 \text{ sec}; \)
\[ P = R_p \times g/C = 0.400 \text{ (for protected + permitted phases, as proposed in this paper)} \]
\[ \text{V}_g = 1200 \text{ vph using Eq. 4, or } 1200/3600 \times 4 = 4/3 \text{ veh/increment} \]
\[ \text{V}_r = 2700 \text{ vph using Eq. 5, or } 2700/3600 \times 4 = 9/3 \text{ veh/increment} \]
\[ g_q = 34.1 \text{ sec using Eq. 10 (measured from the start of the eff. green; or 58.1 sec from red start)} \]
\[ \text{PF}_1 = 1.523 \text{ using Eq. 12} \]
\[ d_1 \times \text{PF}_1 = 15.5 \text{ sec/veh} \]

The results are summarized in tabular form in Table 3, and shown in graphical form in Figure 3.

From Table 3, the first term delay is computed as \( d'_1 = 484/30 = 16.1 \text{ sec/veh} \) and the first term queue is computed as \( Q_1 = \text{MBQ}/\text{lanes} = 30/2 = 15 \text{ veh} \), which are virtually identical to the computed values of \( d_1 \times \text{PF}_1 \) and \( Q_1 \) for the conditions which are approximated by the inputs for the equivalent IQA analysis. The tabulation of trapezoid areas at the bottom of Figure 3 is a more precise calculation which easily accounts for the time of opposing queue dissipation and the actual time of the subject queue dissipation. In this table, \( q_1 \) and \( q_2 \) represent the queue lengths at the start and end of the interval (\( \Delta \)).
Table 3. IQA Method Applied to Platoon Arrivals: Example 3

<table>
<thead>
<tr>
<th>Δ#</th>
<th>time (sec)</th>
<th>#In</th>
<th>#Out</th>
<th>IQA</th>
<th>MBQ</th>
<th>IQAxΔ (partial delay)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-4</td>
<td>9/3</td>
<td>0</td>
<td>9/3</td>
<td>9/3</td>
<td>36/3      effective red phase starts</td>
</tr>
<tr>
<td>2</td>
<td>4-8</td>
<td>9/3</td>
<td>0</td>
<td>18/3</td>
<td>18/3</td>
<td>72/3</td>
</tr>
<tr>
<td>3</td>
<td>8-12</td>
<td>9/3</td>
<td>0</td>
<td>27/3</td>
<td>27/3</td>
<td>108/3</td>
</tr>
<tr>
<td>4</td>
<td>12-16</td>
<td>9/3</td>
<td>0</td>
<td>36/3</td>
<td>36/3</td>
<td>144/3</td>
</tr>
<tr>
<td>5</td>
<td>16-20</td>
<td>9/3</td>
<td>0</td>
<td>45/3</td>
<td>45/3</td>
<td>180/3</td>
</tr>
<tr>
<td>6</td>
<td>20-24</td>
<td>9/3</td>
<td>0</td>
<td>54/3</td>
<td>54/3</td>
<td>216/3</td>
</tr>
<tr>
<td>7</td>
<td>24-28</td>
<td>4/3</td>
<td>4</td>
<td>46/3</td>
<td>58/3</td>
<td>184/3     effective prot. green phase starts</td>
</tr>
<tr>
<td>8</td>
<td>28-32</td>
<td>4/3</td>
<td>4</td>
<td>38/3</td>
<td>62/3</td>
<td>152/3</td>
</tr>
<tr>
<td>9</td>
<td>32-36</td>
<td>4/3</td>
<td>4</td>
<td>30/3</td>
<td>66/3</td>
<td>120/3</td>
</tr>
<tr>
<td>10</td>
<td>36-40</td>
<td>4/3</td>
<td>4</td>
<td>22/3</td>
<td>70/3</td>
<td>88/3</td>
</tr>
<tr>
<td>11</td>
<td>40-44</td>
<td>4/3</td>
<td>3</td>
<td>17/3</td>
<td>74/3</td>
<td>68/3      effective perm. green phase starts</td>
</tr>
<tr>
<td>12</td>
<td>44-48</td>
<td>4/3</td>
<td>3</td>
<td>12/3</td>
<td>78/3</td>
<td>48/3      (opposing g_q = 0.49 sec - ignored)</td>
</tr>
<tr>
<td>13</td>
<td>48-52</td>
<td>4/3</td>
<td>3</td>
<td>7/3</td>
<td>82/3</td>
<td>28/3</td>
</tr>
<tr>
<td>14</td>
<td>52-56</td>
<td>4/3</td>
<td>3</td>
<td>2/3</td>
<td>86/3</td>
<td>8/3</td>
</tr>
<tr>
<td>15</td>
<td>56-60</td>
<td>4/3</td>
<td>2</td>
<td>0</td>
<td>90/3</td>
<td>0         queue dissipates during this interval</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>30</td>
<td>30</td>
<td>121</td>
<td>30</td>
<td>484 veh-sec</td>
</tr>
</tbody>
</table>

This example illustrates that even the new PF\textsubscript{1} formulation proposed in this paper cannot be considered precise when protected-permitted turns are being analyzed, since the saturation flow rate is not constant throughout the green phase (as assumed by the g\textsubscript{q} derivation). An approximation can be made by using the average of the protected and permitted saturation flows, weighted by phase duration. However, that estimate is still not correct since only a portion, if any, of the second phase might be used to dissipate the queue. The IQA method takes this into account precisely, with no additional considerations.

This example also highlights an inconsistency in the HCM2000 protected-permitted procedure in selecting the appropriate $P$ value for the purpose of calculating PF for delay, and PF\textsubscript{2} for queue length. The HCM2000 explicitly states that only the $g/C$ for the protected phase be used to calculate $P$ for use in the PF formula, while it is clearly the intent of the queue method to the use the total protected and permitted phase times. Considering the derivation of $V_g$ and $V_r$ in this paper (which is the foundation of the PF, PF\textsubscript{1} and PF\textsubscript{2} formulations), it is clear that the queue model approach is most consistent with the other assumptions of the method, and should be applied consistently throughout the HCM. With respect to the IQA method, the same assumption should be made (use the total protected and permitted green time) so that the synthesized volumes during the green and red phases are also consistent.
Figure 3. Illustration of IQA Results for Platooned Arrivals, Example 3

Increment $\Delta = 4 \text{ sec}$

- Each "1" is 1/3 vehicle
- Each "2" is 2/3 vehicle
- Each "3" is 3/3 vehicle (each delayed 4 sec)

Display:
```
            r r r r r r g g g g g g g g g g
```

Increment:
```
1 2 3 4 5 6 7 8 9 0 1 2 3 4 5
```

IQA (1/3):
```
1 2 3 4 5 4 3 3 2 1 1
9 8 7 6 5 4 6 8 0 2 7 2 7 2 0
```

**Equivalent Calculations Using Precise Trapezoids**

<table>
<thead>
<tr>
<th>Interval#</th>
<th>$\Delta$(sec)</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$d_1$</th>
<th>MBQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.00</td>
<td>0.00</td>
<td>18.00</td>
<td>216.00</td>
<td>18.00</td>
</tr>
<tr>
<td>2</td>
<td>16.00</td>
<td>18.00</td>
<td>7.33</td>
<td>202.64</td>
<td>23.33</td>
</tr>
<tr>
<td>3</td>
<td>0.49</td>
<td>7.33</td>
<td>7.49</td>
<td>3.63</td>
<td>23.49</td>
</tr>
<tr>
<td>4</td>
<td>17.96</td>
<td>7.49</td>
<td>0.00</td>
<td>67.26</td>
<td>29.48</td>
</tr>
<tr>
<td>5</td>
<td>1.55</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Totals</td>
<td>60.00</td>
<td></td>
<td></td>
<td>489.53</td>
<td>29.48</td>
</tr>
</tbody>
</table>

Results: $d_1=16.3$ $Q_1=14.74$
Discussion

Several questions were raised in the January 2005 white paper [3] regarding the consistency of the proposed IQA method with 1) the first term (Q₁) of the HCM queue model, and 2) the second and third terms (d₂ and d₃) of the HCM delay calculations. It should be apparent from the discussion of this paper, and the numerical examples, that the proposed IQA method is completely consistent with the formulation of Q₁ and that the results of the IQA method can properly be substituted as an alternate, more flexible and less constraining way of evaluating the Q₁ term, exactly in parallel to the proposed replacement of the d₁ evaluation method. This was illustrated with the three example problems above, as well.

The fundamentals of the underlying methods of calculating d₂ and d₃ have also been reviewed, and as expected, these methods are completely compatible with the alternative IQA method and can continue to be used without change as currently documented in [1].

In the January 2005 SigSub white paper [3] a step-by-step process was suggested as a means to execute the trapezoidal IQA method. This process has since been refined, as outlined below:

1. Carry out the analysis for each of the movements of the intersection in order, with through movements first so that the saturation flow rate of permitted left turns can be based on a properly estimated gₘ for opposing through movements.
2. Calculate the capacity in each effective green interval based on the appropriate saturation flow during that interval and sum all capacities in the cycle, including sneakers when applicable. If the total capacity is less than the demand, calculate the inverse v/c ratio and adjust all the inflow rates downward by this reduction factor.
3. Determine the points in the cycle where inflow rates (v) change (ie, due to platooning) and outflow rates (s) change (ie, due to signal changes or opposing-flow conditions). For example, see Figure 3.
4. Start at the end of the effective green for the protected phases of the subject movement (assuming a zero queue at this point).
5. Having determined the queue (q₁) at the end of the previous interval (starting with a queue =0 from step 4), determine the queue (q₂) at the end of the interval by the equation
   \[ q₂ = q₁ + (v - s) \Delta. \]
   If the computed queue is negative (q₂ < 0), determine the time when q₂ = 0 and divide the original time interval into two intervals at this time point, with the second interval having outflow = inflow and delay = 0. For example, see the calculation table at the bottom of Figure 3.
6. Calculate the IQA partial delay during the interval as the area of the trapezoid formed by the queues and flow rates determined by the equation \[ d_i = \Delta (q₁ + q₂) / 2. \]
7. Proceed to step 5 for the next time interval.
Conclusions and Recommendations for Future Research

In summary, the authors offer the following conclusions:

1) The IQA method appears to be fully compatible with HCM2000 theory in handling the first term delay effects of platooned traffic under various signal progression scenarios or arrival types, while offering additional flexibility.

2) It was found in the course of this research that the current HCM progression factor assumes that the queue dissipation time is identical for all progression scenarios as a simplifying assumption. The unsimplified formula for the progression factor has been derived and demonstrated to be consistent with the IQA results.

3) As a result of conclusion (2), the need for further delay adjustments to account for early or late platoon arrivals has been eliminated.

In terms of future research, the highest priority would be (a) to conduct validation studies against field conditions, and (b) to implement and numerically test and evaluate the progression treatment and lane flow allocation model described here within the IQA procedure. This will enable a complete treatment of the HCM delay model, and will bring up the accuracy of its estimates to a level that is commensurate with the HCM users’ expectations.
References


